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Supersymmetry breaking in extended Wess-Zumino Model

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Abstract : Incorporating the Cabibbo-Ferrari field tensor in the vector multiplet and by adding suitable fields, the supersymmetrization of Wess-Zumino model has been carried out and employing the Fayet-Iliopoulos technique of spontaneous supersymmetry breaking, the mass spectrum of the theory has been explained

Keywords : Supersymmetrization, extended Wess-Zumino model, mass spectrum

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1. Introduction

One of the most remarkable ideas in particle physics is the supersymmetry, a symmetry between bosons and fermions [1]. Supersymmetric theories imply that ordinary fields are assembled in multiplets with a consequence that ordinary particles fall into supermultiplets which are mass degenerate [2]. This mass degeneracy forbids the realization of supersymmetry in nature because we do not observe the degenerate Bose-Fermi multiplets In order that the supersymmetry plays a role in nature, it must be spontaneously broken and an appropriate method of supersymmetry breaking has to be found for the supergauge invariant theories in which all particles have the same mass. In this regard, Fayet and Iliopoulos [3] have suggested a unique model with a spontaneously broken supergauge symmetry which combines supergauge invariance with ordinary gauge invariance and shows that the gauge boson acquires a mass as a result of Higgs mechanism. In the present paper, we apply the similar technique to a supersymmetric model which is the extension of Wess-Zumino [4] model in that the photon field of the vector multiplet has the form [5–7]

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$$C_{\mu\nu} = \partial_\mu v_\nu - \partial_\nu v_\mu - \frac{1}{2}\delta_{\mu\nu\rho\sigma}(\partial^\rho w^\sigma - \partial^\sigma w^\rho) \tag{1}$$

instead of the usual

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

and show by adding suitable fields the mechanism of spontaneous breaking of supersymmetry may be initiated which creates a difference in the scalar and spinor masses. A typical situation is also shown to occur in which ordinary gauge invariance also breaks down resulting in the generation of vector field mass and non-diagonalizing the fermion mass which has been shown to be diagonalized by Fayet Iliopoulos [3] transformations. The significance of eq. (1) lies in the fact that it adds new dimensions to the theory as the tensor (1) is suitable to formulate the Abelian gauge theories with both electric and magnetic sources [8–10].

2. The Lagrangian

Since in the present case, two massless vector fields v_μ and w_μ are present, supergauge invariance would require the incorporation of additional scalar and spinor fields. The Lagrangian density in accordance with the Wess-Zumino model [4] would then be

$$\begin{aligned}
L = & -\frac{1}{2}\sum_{i=1}^{4} \partial_\mu A_i \partial^\mu A_i + \partial_\mu B_i \partial^\mu B_i - F_i^2 - C_i^2 + i\overline{\psi}_i \partial\psi_i \\
& - 2m\left(F_i A_i + C_i B_i - (i/2)\overline{\psi}_i \psi_i\right) - \frac{1}{4}C_{\mu\nu}C^{\mu\nu} \\
& - \frac{1}{2}\overline{\lambda}_1 \partial\lambda_1 - \frac{1}{2}\overline{\lambda}_2 \partial\lambda_2 + \frac{1}{2}D_1^2 + \frac{1}{2}D_2^2 \\
& + e\left[D_1\left(A_1B_2 - A_2B_1\right) - V_\mu\left(A_1\partial^\mu A_2 - A_2\partial^\mu A_1 + B_1\partial^\mu B_2 - B_2\partial^\mu B_1 \right.\right. \\
& \left.\left. \qquad - i\overline{\psi}_1\gamma^\mu\psi_2\right)\right] \\
& + g\left[D_2\left(A_3B_4 - A_4B_3\right) - w_\mu\left(A_3\partial^\mu A_4 - A_4\partial^\mu A_3 + B_3\partial^\mu B_4 - B_4\partial^\mu B_3 \right.\right. \\
& \left.\left. \qquad - i\overline{\psi}_3\gamma^\mu\psi_4\right)\right] \\
& - ie\overline{\lambda}_1\left[\left(A_1 + \gamma_5 B_1\right)\psi_2 - \left(A_2 + \gamma_5 B_2\right)\psi_1\right] - ig\overline{\lambda}_2\left[\left(A_3 + \gamma_5 B_3\right)\psi_4 \right. \\
& \left. \qquad - \left(A_4 + \gamma_5 B_4\right)\psi_3\right] \\
& - \frac{1}{2}e^2 v_\mu^2\left(A_1^2 + A_2^2 + B_1^2 + B_2^2\right) - \frac{1}{2}g^2 w_\mu^2\left(A_3^2 + A_4^2 + B_3^2 + B_4^2\right),
\end{aligned} \tag{3}$$

where subscript i indicates the fields 1, 2, 3, 4; A_i are the scalar fields, B_i the pseudoscalar, y_i the massive spinor; l_1, l_2 the massless spinor; F_i, C_i and D_1, D_2 are the auxillary fields in which F_i indicates the massive character of A_i and C_i those of B_i; $\not\partial = \gamma^\mu \partial_\mu$, γ^μ and γ_5 are the Dirac matrices; e and g are two coupling parameters in which e couples only the fields v_μ, A_1, A_2, B_1, B_2, ψ_1, ψ_2 and λ_1 while g only the fields w_μ, A_3, A_4, B_3, B_4, ψ_3, ψ_4 and λ_2. The presence of two coupling constants does not violate the concept of supergauge invariance because, as indicated above, all trilinear and quadrilinear interaction terms of one kind couple through e and those of other through g.

In order to see the ordinary and supergauge invariance of the Lagrangian density (3), we may write the general gauge transformations as

$$\phi_i \rightarrow \phi_i + \delta\phi_i, \tag{4}$$

where ϕ indicates the fields in the theory. The infinitesimal transformations for ordinary gauge are [4]

$$\delta v_\mu = \partial_\mu \Lambda, \tag{5a}$$

$$\delta w_\mu = \partial_\mu \Gamma, \tag{5b}$$

$$\delta\phi_i = a e \Lambda \phi_{i+a}, \tag{5c}$$

$$\delta\phi_j = b g \Gamma \phi_{j+b}, \tag{5d}$$

$$\delta\lambda_i = 0, \tag{5e}$$

$$\delta D_i = 0, \tag{5f}$$

where $i = 1, 2$; $j = 3, 4$; Λ and Γ are the scalar parameter functions, ϕ denotes the fields A, B and ψ

$$a = (-1)^{i+1}, \qquad b = (-1)^{j+1}. \tag{5g}$$

For the supergauge transformations we have for $i = 1, 2$

$$\begin{aligned}
\delta A_i &= i\bar{\varepsilon}\psi_i, \\
\delta B_i &= i\bar{\varepsilon}\gamma_5\psi_i, \\
\delta\psi_i &= \partial_\mu\left(A_i - \gamma_5 B_i\right)\gamma^\mu\varepsilon + \left(F_i + \gamma_5 C_i\right)\varepsilon, \\
\delta F_i &= i\bar{\varepsilon}\gamma^\mu\partial_\mu\psi_i, \\
\delta G_i &= i\bar{\varepsilon}\gamma_5\gamma^\mu\partial_\mu\psi_i, \\
\delta v_\mu &= i\bar{\varepsilon}\gamma_\mu\lambda_1, \\
\delta\lambda_1 &= -\frac{1}{2}G_{\mu\nu}\gamma^\mu\gamma^\nu + D_1\gamma_5\varepsilon, \\
\delta D_1 &= i\bar{\varepsilon}\gamma_5\gamma^\mu\partial_\mu\lambda_1,
\end{aligned} \tag{6}$$

where ε is an x-independent Grassmann variable and bar denotes adjoint.

For $i = 3, 4$ the supergauge transformations are similar to eq. (6) except for the following replacements

$$v_\mu \rightarrow w_\mu, \quad D_1 \rightarrow D_2, \quad \lambda_1 \rightarrow \lambda_2 \quad \text{and} \quad G_{\mu\nu} \rightarrow \tilde{G}_{\mu\nu} \tag{7}$$

where $\tilde{G}_{\mu\nu}$ is the dual of eq. (1)

$$\tilde{G}_{\mu\nu} = \partial_\mu w_\nu - \partial_\nu w_\mu + \frac{1}{2}\delta_{\mu\nu\rho\sigma}\left(\partial^\rho v^\sigma - \partial^\sigma v^\rho\right). \tag{8}$$

The field equations of the theory may be obtained (Appendix-A) through the Euler-Lagrange variation of the Lagrangian density (3) which when used in the variation δL of L give

$$\delta L = \partial_\mu k^\mu, \tag{9}$$

where

$$\begin{aligned} k^\mu = {} & \delta A_1\left(-\partial^\mu A_1 + ev^\mu A_2\right) - \delta A_2\left(\partial^\mu A_2 + ev^\mu A_1\right) \\ & + \delta A_3\left(-\partial^\mu A_3 + gw^\mu A_4\right) - \delta A_4\left(\partial^\mu A_4 + gw^\mu A_3\right) \\ & + \delta B_1\left(-\partial^\mu B_1 + ev^\mu B_2\right) - \delta B_2\left(\partial^\mu B_2 + ev^\mu B_1\right) \\ & + \delta B_3\left(-\partial^\mu B_3 + gw^\mu B_4\right) - \delta B_4\left(\partial^\mu B_4 + gw^\mu B_3\right) \\ & - (1/2)\sum_{i=1}^{4} \overline{\psi}_i \gamma^\mu \delta\psi_i \end{aligned} \tag{10}$$

in which δA_1 *etc.* are the infinitesimal variations in A_1 *etc.* and terms with other variations which do not appear in eq. (10) vanish. These variations for ordinary gauge transformations are given by eq. (5) and for supergauge transformations by eq. (6). It is easy to observe that the eq. (9) will be satisfied for both eqs. (5) and (6) thus from eq. (9) using Gause theorem, the action

$$\int d^4x\,\delta L = \int_R d^4x\,\partial_\mu k^\mu(x) = \int_S d\sigma_\mu(x) k^\mu(x) \tag{11}$$

with R denoting the arbitrary four volume and S the [11] arbitrary closed hypersurface that is the boundary of R. $d\sigma_\mu(x)$ is the infinitesimal hypersurface element. For a hypersurface sufficiently far away from where the fields are non-zero, the last integral of eq. (11) will vanish and thus leave the action invariant under both the ordinary and supergauge transformations.

3. The supersymmetry breaking

(a) Generation of scalar-spinor mass difference :

From the gauge transformations (5*f*), we observe that

$$\delta_G D_i = 0 \tag{12a}$$

and from the supergauge transformations (6),

$$\delta_S D_i = \left(i\bar{\varepsilon}\gamma_5\gamma^\mu\partial_\mu\lambda_i\right), \tag{12b}$$

where G and S stand for gauge and supergauge respectively. Eq. (12b) indicates that the supergauge variation of D_i ($i = 1, 2$) transforms as a density. Thus any term proportional to D_i included in the already supergauge invariant Lagrangian density, will not break the ordinary as well as supergauge invariance, it will however, trigger [3] the spontaneous breaking of supersymmetry. The choice of D_i is because of its lowest dimensionality. Adding therefore the terms proportional to D_1 and D_2, the modified Lagrangian density becomes

$$L_{\text{mod}} = L + \xi_1 D_1 + \xi_2 D_2, \tag{13}$$

where ξ_1 and ξ_2 are the constants of proportionality and L has the form (3). With the Lagrangian density (13) the field equations are same as in appendix, except that for D_1 and D_2 they (eqs. A23, A24) are modified as

$$D_1 = -e\left(A_1B_2 - A_2B_1\right) - \xi_1, \tag{14a}$$

and

$$D_2 = -g\left(A_3B_4 - A_4B_3\right) - \xi_2. \tag{14b}$$

We may now identify the scalar potential U from the Lagrangian density (13) as the part which does not contain derivatives or fermions [12]. Using eq. (3) in (13) it is

$$\begin{aligned} U = {} & \frac{1}{2}\sum_{i=1}^{4}\left[F_i^2 + G_i^2 + mF_iA_i + mG_iB_i\right] \\ & + \frac{1}{2}D_1^2 + eD_1\left(A_1B_2 - A_2B_1\right) + \xi_1 D_1 \\ & + \frac{1}{2}D_2^2 + gD_2\left(A_3B_4 - A_4B_3\right) + \xi_2 D_2, \end{aligned} \tag{15}$$

which on the further use of field eqs. (14), (A10) and (A11) acquires the form

$$\begin{aligned} U = {} & -\frac{1}{2}m^2\sum_{i=1}^{4}\left(A_i^2 + B_i^2\right) - \xi_1 e\left(A_1B_2 - A_2B_1\right) - \xi_2 g\left(A_3B_4 - A_4B_3\right) \\ & - \frac{1}{2}\left[e^2\left(A_1B_2 - A_2B_1\right)^2 + g^2\left(A_3B_4 - A_4B_3\right)^2\right] - \frac{1}{2}\left(\xi_1^2 + \xi_2^2\right). \end{aligned} \tag{16}$$

It is obvious from eq. (16) that U has a non-diagonal scalar mass matrix, which may be diagonalized by introducing the following transformations [3]

$$\begin{aligned} \tilde{A}_1 &= \frac{1}{\sqrt{2}}(A_1 + B_2), & \tilde{A}_2 &= \frac{1}{\sqrt{2}}(A_2 + B_1), \\ \tilde{A}_3 &= \frac{1}{\sqrt{2}}(A_3 + B_4), & \tilde{A}_4 &= \frac{1}{\sqrt{2}}(A_4 + B_3), \\ \tilde{B}_1 &= \frac{1}{\sqrt{2}}(B_1 - A_2), & \tilde{B}_2 &= \frac{1}{\sqrt{2}}(B_2 - A_1), \\ \tilde{B}_3 &= \frac{1}{\sqrt{2}}(B_3 - A_4), & \tilde{B}_4 &= \frac{1}{\sqrt{2}}(B_4 - A_3), \end{aligned} \tag{17}$$

which give

$$
\begin{aligned}
A_1B_2 - A_2B_1 &= \frac{1}{2}\left(\tilde{A}_1^2 + \tilde{B}_1^2 - \tilde{A}_2^2 - \tilde{B}_2^2\right), \\
A_3B_4 - A_4B_3 &= \frac{1}{2}\left(\tilde{A}_3^2 + \tilde{B}_3^2 - \tilde{A}_4^2 - \tilde{B}_4^2\right), \\
A_1^2 + A_2^2 + B_1^2 + B_2^2 &= \tilde{A}_1^2 + \tilde{A}_2^2 + \tilde{B}_1^2 + \tilde{B}_2^2, \\
A_3^2 + A_4^2 + B_3^2 + B_4^2 &= \tilde{A}_3^2 + \tilde{A}_4^2 + \tilde{B}_3^2 + \tilde{B}_4^2.
\end{aligned} \tag{18}
$$

Using eqs. (18) in (16) we get

$$
\begin{aligned}
\tilde{U} = &-\frac{1}{2}\left(m^2 + \xi_1 e\right)\left(\tilde{A}_1^2 + \tilde{B}_1^2\right) - \frac{1}{2}\left(m^2 + \xi_2 g\right)\left(\tilde{A}_3^2 + \tilde{B}_3^2\right) \\
&- \frac{1}{2}\left(m^2 - \xi_1 e\right)\left(\tilde{A}_2^2 + \tilde{B}_2^2\right) - \frac{1}{2}\left(m^2 - \xi_2 g\right)\left(\tilde{A}_4^2 + \tilde{B}_4^2\right) \\
&- \frac{1}{B}\left[e^2\left(\tilde{A}_1^2 - \tilde{A}_2^2 + \tilde{B}_1^2 - \tilde{B}_2^2\right) + g^2\left(\tilde{A}_3^2 - \tilde{A}_4^2 + \tilde{B}_3^2 - \tilde{B}_4^2\right)\right] \\
&- \frac{1}{2}\left(\xi_1^2 + \xi_2^2\right).
\end{aligned} \tag{19}
$$

In eq. (19), if $(m^2 - x_1 e)$ and $(m^2 - x_2 g)$ have positive values the minimum of potential which occurs at

$$\left\langle \tilde{A}_i \right\rangle = 0 = \left\langle \tilde{B}_i \right\rangle \tag{20a}$$

has a value

$$\left\langle \tilde{U} \right\rangle = \frac{1}{2}\left(\xi_1^2 + \xi_2^2\right). \tag{20b}$$

From these equations, we observe that the expectation value of all the fields in eq. (19) is zero while that of potential is not zero. This shows that the supersymmetry is spontaneously broken while for ordinary gauge invariance we are led by the transformations (5c–d) and (17) that it remains intact. We may read the mass spectrum from eq. (19) as

$$
\begin{aligned}
{}^m\tilde{A}_1 &= {}^m\tilde{B}_1 = \left(m^2 + \xi_1 e\right)^{1/2}, \\
{}^m\tilde{A}_2 &= {}^m\tilde{B}_2 = \left(m^2 - \xi_1 e\right)^{1/2}, \\
{}^m\tilde{A}_3 &= {}^m\tilde{B}_3 = \left(m^2 + \xi_2 g\right)^{1/2}, \\
{}^m\tilde{A}_4 &= {}^m\tilde{B}_4 = \left(m^2 - \xi_2 g\right)^{1/2}, \\
{}^m\psi_i &= m, \quad {}^m\lambda_j = mv_\mu = mw_\mu = 0.
\end{aligned} \tag{21}
$$

Eqs. (21) clearly shows that the spontaneous breaking of supersymmetry induced by the introduction of the term $\xi_i D_i$, has generated a mass difference between the scalars and spinors. The massless spinors λ_1 and λ_2 are the goldstone fermions.

(b) Generation of vector and spinor field mass :

In eq. (19), if $(m^2 - \xi_1 e)$ and $(m^2 - \xi_2 g)$ have negative values, the minimum of the potential occurs at a finite value for the matter fields. In this case also the minimum $<\tilde{U}>$, from eq. (19), is obviously not zero, the exact value depends on our choices of $\langle \tilde{A}_i \rangle$ and $\langle \tilde{B}_i \rangle$ with $\langle \tilde{A}_2 \rangle$, $\langle \tilde{A}_4 \rangle$ or $\langle \tilde{B}_2 \rangle$, $\langle \tilde{B}_4 \rangle$ not vanishing. A non-zero minimum value of $\langle \tilde{U} \rangle$ again tells that here also the supersymmetry is broken. However, unlike the previous case, the ordinary gauge invariance is also broken which manifests itself in generating mass for vector fields and non-diagonalizing the mass of spinor fields. Explicitly, the relevant terms are contained in the following expression

$$
\begin{aligned}
&\tilde{U} - \frac{1}{2} e^2 v_\mu^2 \left(\tilde{A}_1^2 + \tilde{A}_2^2 + \tilde{B}_1^2 + \tilde{B}_2^2\right) - \frac{1}{2} g^2 w_\mu \left(\tilde{A}_3^2 + \tilde{A}_4^2 + \tilde{B}_3^2 + \tilde{B}_4^2\right) \\
&- \frac{ie\bar{\lambda}_1}{\sqrt{2}}\left[\left(\tilde{A}_1 + \gamma_5 \tilde{A}_2\right)\psi_2 - \left(\tilde{A}_2 + \gamma_5 \tilde{A}_1\right)\psi_1\right] \\
&\qquad - \frac{ie\bar{\lambda}_1}{\sqrt{2}}\left[\left(\tilde{B}_2 - \gamma_5 \tilde{B}_1\right)\psi_2 - \left(\tilde{B}_1 - \gamma_5 \tilde{B}_2\right)\psi_1\right] \\
&\frac{ig\bar{\lambda}_2}{\sqrt{2}}\left[\left(\tilde{A}_3 + \gamma_5 \tilde{A}_4\right)\psi_4 - \left(\tilde{A}_4 + \gamma_5 A_3\right)\psi_1\right. \\
&\qquad - \frac{ig\bar{\lambda}_2}{\sqrt{2}}\left[\left(\tilde{B}_4 - \gamma_5 \tilde{B}_3\right)\psi_4 - \left(\tilde{B}_3 - \gamma_5 B_4\right)\psi_3\right]
\end{aligned}
\tag{22}
$$

in which the first term $(\tilde{U})$ is as in eq. (19) and rest are the tilde (~) transforms (eq. 17) of the corresponding terms in the Lagrangian density (eq. 3). Looking again at eq. (19), we observe that the coefficient of $(m^2 - \xi_1 e)$ is symmetric in $\tilde{A}_2$ and $\tilde{B}_2$ while that of $(m^2 - \xi_2 g)$ in $\tilde{A}_4$ and $\tilde{B}_4$. So for simplicity, we may take variations of $\tilde{U}$ only with respect to $\tilde{A}_2$ and $\tilde{A}_4$ respectively to obtain the respective minimum values. The minimum $\tilde{U}$ then occurs at [13]

$$
\tilde{A}_2^2 = a^2 = \frac{2}{e^2}\left(\xi_1 e - m^2\right) \tag{23a}
$$

and
$$
\tilde{A}_4^2 = b^2 = \frac{2}{g^2}\left(\xi_2 g - m^2\right) \tag{23b}
$$

Thus, in $<\tilde{U}>$ *versus* $<\hat{A}_2>$ plot, the minima may be made to coincide with the origin by a translation

$$\tilde{A}_2 \to \tilde{A}_2 + a \tag{24a}$$

and in $<\tilde{U}>$ *versus* $<\tilde{A}_4>$ plot by the translation

$$\tilde{A}_4 \to \tilde{A}_4 + b. \tag{24b}$$

For other fields in eq. (19) however, a and b vanish such that for $\tilde{A}_1, \tilde{B}_1$ and $\tilde{B}_2$

$$\xi_1 e = m^2 \tag{25a}$$

and for $\tilde{A}_3, \tilde{B}_3$ and $\tilde{B}_4$

$$\xi_2 g = m^2. \tag{25b}$$

Now expanding $\tilde{U}$, according to eqs. (24) in the form

$$\tilde{U}(a+\tilde{A}_2) = \tilde{U}(a) + \tilde{A}_2\left(\frac{d\tilde{U}}{d\tilde{A}_2}\right) + \frac{\tilde{A}_2^2}{2!}\left(\frac{d^2\tilde{U}}{d\tilde{A}_2^2}\right) + \ldots \tag{26}$$

with similar relation for $\tilde{U}(b+\tilde{A}_4)$, and using eqs. (23), (25) and (22), the mass terms in the Lagrangian acquire the form

$$\begin{aligned}
&-\frac{1}{2}(2m^2)\left(\tilde{A}_1^2 + \tilde{B}_1^2 + \tilde{A}_3^2 + \tilde{B}_3^2\right) - \frac{1}{2}\left(e^2a^2\right)\tilde{A}_2^2 - \frac{1}{2}\left(g^2b^2\right)\tilde{A}_4^2 \\
&-\frac{1}{2}\left(e^2a^2\right)v_\mu^2 \mp \frac{1}{2}\left(g^2b^2\right)w_\mu^2 - \frac{ie}{2}\sqrt{2}\bar{\lambda}_1(\gamma_5\psi_2 - \psi_1) \\
&-\frac{1}{2}g\sqrt{2}\bar{\lambda}_2(\gamma_5\psi_4 - \psi_3) - \frac{1}{2}m\sum_{i=1}^{4}\bar{\psi}_i\psi_i,
\end{aligned} \tag{27}$$

which sets the mass spectrum as

$$\begin{aligned}
&{}^m\tilde{A}_1 = {}^m\tilde{B}_1 = {}^m\tilde{A}_3 = {}^m\tilde{B}_3 = m\sqrt{2}, \\
&{}^m v_\mu = {}^m\tilde{A}_2 = ea, \\
&{}^m w_\mu = {}^m\tilde{A}_4 = gb, \\
&{}^m\tilde{B}_2 = {}^m\tilde{B}_4 = {}^m\bar{\lambda}_j = 0.
\end{aligned} \tag{28}$$

We may observe from eqs. (27) and (28) that v_μ and w_μ have acquired mass while $\tilde{B}_2$ and $\tilde{B}_4$ have turned massless. $\tilde{B}_2$ and $\tilde{B}_4$ are therefore the Goldstone bosons in the process of breaking of ordinary gauge invariance. The difference appearing in the scalar masses is the result of supersymmetry breaking. The fermion masses in eq. (27) have now become non-diagonal which can be diagonalized by using the transformations [3] $\eta_1 \ldots\ldots\ldots \eta_6$, where $$\eta_1 = \frac{1}{2}\left[\left(1 + \cos\theta_1\right)\psi_1 - (1 - \cos\theta_1)\gamma_5\psi_2 - \sqrt{2}\sin\theta_1\lambda_1\right], \tag{29a}$$

$$\eta_2 = \frac{1}{2}\left[\left(1 - \cos\theta_1\right)\gamma_5\psi_2 + (1 + \cos\theta_1)\psi_2 + \sqrt{2}\sin\theta_1\gamma_5\lambda_1\right], \tag{29b}$$

$$\eta_3 = \frac{1}{\sqrt{2}}\sin\theta_1\left(\psi_1 + \gamma_5\psi_2\right) + \cos\theta\lambda_1, \tag{29c}$$

with $$\tan\theta_1 = \frac{ea}{m} \tag{29d}$$

and η_4, η_5, η_6 can be obtained from eq. (29) by changing

$$\psi_1 \to \psi_3, \psi_2 \to \psi_4, \theta_1 \to \theta_2, \lambda_1 \to \lambda_2, e \to g \text{ and a} \to b. \tag{30}$$

A substitution of eqs. (29) and (30) into the expression (27) leads to the following diagonal form for the spinor mass

$$-\frac{1}{2}i\left[\left(m^2 + a^2e^2\right)^{1/2}\left(\bar{\eta}_1\eta_1 + \bar{\eta}_2\eta_2\right) + \left(m^2 + b^2g^2\right)^{1/2}\left(\bar{\eta}_4\eta_4 + \bar{\eta}_5\eta_5\right) \tag{31}$$

which indicates that the spinors η_3 and η_6 have turned massless. Recalling the discussion below eq. (21) we observe that the Goldstone fermion λ_1 is identified with η_3 and λ_2 with η_6.

4. Discussion

We have considered the breaking of supersymmetry by introducing the Fayet Iliopoulos terms in the Lagrangian density. The phenomena is similar to Abelian Higgs model [14–17] in which the spontaneous breaking of local $U(1)$ symmetry takes place by a vacuum expectation value of a scalar field. Though any auxillary field F_i, G_i or D_i can be used to provide the necessary terms for inducing spontaneous breaking of supersymmetry, it is the D field which is chosen because of its lowest dimensionality. The nonzero vacuum expectation value of the scalar potential (eq. 19) has indicated the breaking of supersymmetry as a result of which the new tilde scalars have acquired masses $(m^2 \pm \xi_1 e)^{1/2}$ and $(m^2 \pm \xi_2 g)^{1/2}$ while the spinors in the Lagrangian density (3) continue to have mass m and v_μ, w_μ and λ remain massless. Thus a difference in spinor-scelar masses has emerged because of supersymmetry breaking. In the particular case of $\xi_1 e$, $\xi_2 g$ being greater than m, the breaking of both supersymmetry and the gauge symmetry has been shown to occur as a result of which the mass spectrum has become as in eq. (28), where for example, the scalar $\tilde{A}_1$ has mass $m\sqrt{2}$ and $\tilde{A}_2$ has $e\,a$. This mass difference is because of supersymmetry breaking while the masses $m_{v_\mu} = ea$, $m_{w_\mu} = gb$ have generated because of breaking of ordinary gauge symmetry in which scalars $\tilde{B}_2$ and $\tilde{B}_4$ emerge as Goldstone bosons. A significant feature of the analysis is that the trans formations (29) which diagonalize the fermion mass matrix of the expression (27), identify the Goldstinoes $\lambda_1 = \eta_3$, $\lambda_2 = \eta_6$ in the breaking of supersymmetry.

References

[1] E Witten *Nucl. Phys* **B202** 252 (1982)

[2] S Ferrar *Supersymmetry* (Singapore : World Scientific) (1987)

[3] P Fayet and J Ilipoulos *Phys. Lett* **51B** 461 (1974)

[4] J Wess and B Zumino *Nucl. Phys.* **878** 1 (1974)

[5] N Cabibbo and E Ferrari *Nuovo Cim.* **23** 1147 (1962)

[6] B P Bhatt and D C Joshi *Indian J. Pure Appl. Phys* **32** 8 (1994)

[7] B P Bhatt, K N Beramolla, D C Joshi and B S Rajput *Indian J. Pure Appl. Phys.* **32** 371 (1994)

[8] M P Benjwal and D C Joshi *Indian J Pure Appl. Phys* **24** 213 (1986)

[9] M P Benjwal and D C Joshi *Phys. Rev.* **D36** 629 (1987)

[10] D C Joshi and B S Rajput *Hadron J.* **4** 1805 (1981)

[11] P Roman *Introduction to Quantum Field Theory* (New York : John Willey) 9 (1969)

[12] H P Nilles *Phys. Rep.* **110** 17 (1984)

[13] C Itzykson and J B Zuber *Quantum Field Theory* (New York : McGraw Hill) 522 (1986)

[14] G Woo *Phys. Rev.* **D16** 1014 (1977)

[15] M Vargas and J L Lucio M *IV Mexical School of Particles and Fields* (Maxico, 3–14 Dec) (1990)

[16] P Valtancoli *Int. J. Mod. Phys.* **7A** 4335 (1992)

[17] D Bezeia *Phys. Rev.* **D46** 1879 (1992)

Appendix

Applying the Euler-Lagrange equations

$$\frac{\partial L}{\partial \phi} - \partial_\mu \frac{L}{\partial \partial_\mu \phi} = 0 \tag{A1}$$

to the Lagrangian density (3), we obtain the following field equations. ϕ in eq. (A1) represents the various fields in the Lagrangian density.

$$\partial_\mu \partial^\mu A_1 + mF_1 = eA_2 \partial_\mu v^\mu + 2ev_\mu \partial^\mu A_2 - eD_1 B_2 + ie\bar{\lambda}_1 \psi_2 + e^2 v_\mu v^\mu A_1, \tag{A2}$$

$$\partial_\mu \partial^\mu A_2 + mF_2 = -eA_1 \partial_\mu v^\mu - 2ev_\mu \partial^\mu A_1 + eD_1 B_1 - ie\bar{\lambda}_1 \psi_1 + e^2 v_\mu v^\mu A_2, \tag{A3}$$

$$\partial_\mu \partial^\mu A_3 + mF_3 = gA_4 \partial_\mu w^\mu + 2gw_\mu \partial^\mu A_4 - gD_2 B_4 + ig\bar{\lambda}_2 \psi_4 + g^2 w_\mu w^\mu A_3, \tag{A4}$$

$$\partial_\mu \partial^\mu A_4 + mF_4 = -gA_3 \partial_\mu w^\mu - 2gw_\mu \partial^\mu A_4 + gD_2 B_3 - ig\bar{\lambda}_2 \psi_3 + g^2 w_\mu w^\mu A_4, \tag{A5}$$

$$\partial_\mu \partial^\mu B_1 + mG_1 = eB_2 \partial_\mu v^\mu + 2ev_\mu \partial^\mu B_2 + eD_1 A_2 + ie\bar{\lambda}_1 \gamma_5 \psi_2 + e^2 v_\mu v^\mu B_1, \tag{A6}$$

$$\partial_\mu \partial^\mu B_2 + mG_2 = -eB_1 \partial_\mu v^\mu - 2ev_\mu \partial^\mu B_1 - eD_1 A_1 - ie\bar{\lambda}_1 \gamma_5 \psi_1 + e^2 v_\mu v^\mu B_2, \tag{A7}$$

$$\partial_\mu \partial^\mu B_3 + mG_3 = gB_4 \partial_\mu w^\mu + 2ew_\mu \partial^\mu B_4 + gD_2 A_4 + ig\bar{\lambda}_2 \gamma_5 \psi_4 + g^2 w_\mu w^\mu B_3, \tag{A8}$$

$$\partial_\mu \partial^\mu B_4 + mG_4 = -gB_4 \partial_\mu w^\mu - 2gw_\mu \partial^\mu B_3 - gD_2 A_3 - ig\bar{\lambda}_2 \gamma_5 \psi_3 + g^2 w_\mu w^\mu B_4, \tag{A9}$$

$$F_i = -mA_i, \tag{A10}$$

$$G_i = -mB_i, \tag{A11}$$

$$i = 1,2,3,4,$$

$$\partial_\mu\left(\overline{\psi}_1\gamma^4\right) - m\overline{\psi}_1 = -2e\lambda_1\left(A_2 + \gamma_5 B_2\right), \tag{A12}$$

$$\partial_\mu\left(\overline{\psi}_2\gamma^\mu\right) - m\overline{\psi}_2 = 2e\overline{\lambda}_1\left(A_1 + \gamma_5 B_1\right) - 2ev_\mu\overline{\psi}_1\gamma^\mu, \tag{A13}$$

$$\partial_\mu\left(\overline{\psi}_3\gamma^\mu\right) - m\overline{\psi}_3 = 2g\lambda_2\left(A_4 + \gamma_5 B_4\right), \tag{A14}$$

$$\partial_\mu\left(\overline{\psi}_4\gamma^\mu\right) - m\overline{\psi}_4 = 2g\overline{\lambda}_2\left(A_3 + \gamma_5 B_3\right) - 2gw_\mu\overline{\psi}_3\gamma^\mu, \tag{A15}$$

$$\gamma^\mu\partial_\mu\psi_1 + m\psi_1 = 2ev_\mu\gamma^\mu\psi_2, \tag{A16}$$

$$\gamma^\mu\partial_\mu\psi_2 + m\psi_2 = 0, \tag{A17}$$

$$\gamma^\mu\partial_\mu\psi_3 + m\psi_3 = 2gw_\mu\gamma^\mu\psi_4, \tag{A18}$$

$$\gamma^\mu\partial_\mu\psi_4 + m\psi_4 = 0, \tag{A19}$$

$$\overline{\lambda}_j\gamma^\mu = 0, \quad j = 1,2. \tag{A20}$$

$$\gamma^\mu\partial_\mu\lambda_1 = 2e\left[(A_2 + \gamma_5 B_2)\psi_1 - (A_1 + \gamma_5 B_1)\psi_2\right], \tag{A21}$$

$$\gamma^\mu\partial_\mu\lambda_2 = eg\left[(A_4 + \gamma_5 B_4)\psi_3 - (A_3 + \gamma_5 B_3)\psi_4\right], \tag{A22}$$

$$D_1 = -e\left(A_1 B_2 - A_2 B_1\right), \tag{A23}$$

$$D_2 = -g\left(A_3 B_4 - A_4 B_3\right), \tag{A24}$$

$$\begin{aligned}\partial^\nu G_{\mu\nu} = {} & eA_1\partial_\mu A_2 - eA_2\partial_\mu A_1 + eB_1\partial_\mu B_2 - eB_2\partial_\mu B_1 - ie\overline{\psi}_1\gamma_\mu\psi_2 \\ & + e^2 v_\mu\left(A_1^2 + A_2^2 + B_1^2 + B_2^2\right)\end{aligned} \tag{A25}$$

$$\begin{aligned}\partial^\nu \tilde{G}_{\mu\nu} = {} & gA_3\partial_\mu A_4 - gA_4\partial_\mu A_3 + gB_3\partial_\mu B_4 - gB_4\partial_\mu B_3 - ig\overline{\psi}_3\gamma_\mu\psi_4 \\ & + g^2 w_\mu\left(A_3^2 + A_4^2 + B_3^2 + B_4^2\right)\end{aligned} \tag{A26}$$

where $\tilde{G}_{\mu\nu}$ is given by eq. (8).